Mead

COMPOSITION

LOGBOOK # 60

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GHOST IN THE MACHINE

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ghost in the machine

LOGBOOK #60

1999 341 .. 2000 041

Michael William Hentrich

Logbook 60

contents

Come With Me

(To The Bitter Depths of My Soul)

COGITATION # Ø44 Faradays Law of Induction

induced ~~e~~ emf $\mathcal{E}$ depends on change in magnetic field

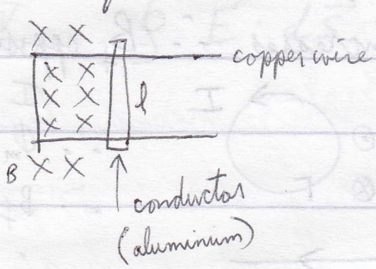
induced emf $\mathcal{E}$ also depends on the area of the loop

$$\mathcal{E} = -N\frac{d\Phi_m}{dt}$$

induced emf — number of loops — Φ_m → magnetic flux — $\frac{d\Phi_m}{dt}$ → derivative of magnetic flux's rate of change

$$\Phi_m = \int \vec{B}\cdot d\vec{A} \Rightarrow BA\cos\theta$$

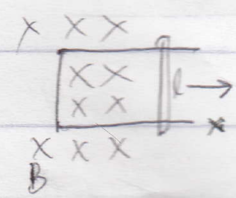
if B, A, or θ changes, then Φ_m changes



NO emf

but, when l moves → area changes, Φ_m changes

∴ emf induced (MOTIONAL emf)

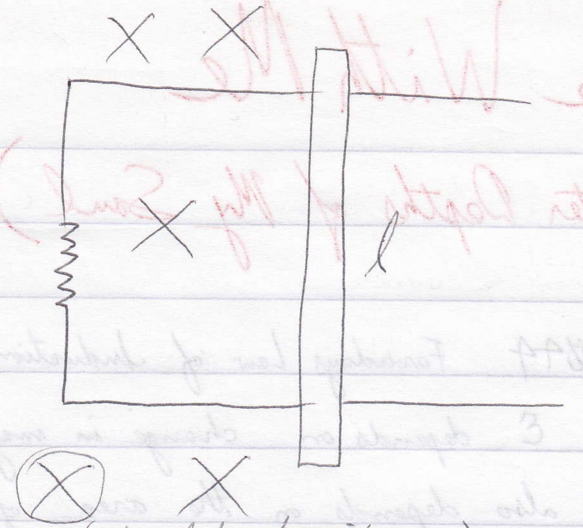


$$\Phi_m = Blx$$

$$\mathcal{E} = N\frac{d\Phi_m}{dt} = N\frac{d(Blx)}{dt} = NBl\frac{dx}{dt} = NBlv$$

if $v \perp B$

COGITATION #∅45 $\boxed{\emptyset}$ LENZ's LAW

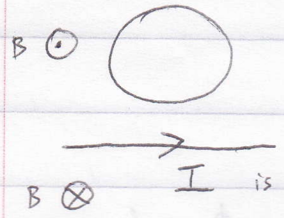


B (original direction into paper)

current (moving charge) is the source of B (magnetic field)

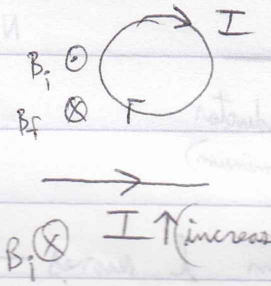
If Φ_m (magnetic flux) is increased, induced B is opposite original B. If Φ_m is decreased, induced B is same as original.

example: if area increases, Φ_m increases $\therefore$ B_f opposite B_i



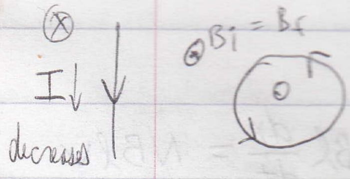
I is constant

$\therefore$ No induced current because Φ_m is not changed



I ↑ (increases)

I increases
$\therefore$ Φ_m increases
$\therefore$ B_f opposite
$\therefore$ current in loop is CW



I induced and CCW

COGITATION #046 Induced emf and electric field 3

recap: Faraday's Law: $\mathcal{E} = -N \dfrac{d\Phi_m}{dt}$

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{s} = -N \frac{d\Phi_m}{dt}$$

$V = Ed$ only if E is constant.

Maxwell's Equations cover everything in Electromagnetism:

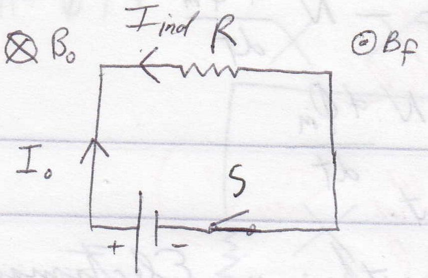
I. $\oint \vec{E} \cdot d\vec{A} = \dfrac{q_{in}}{\epsilon_0}$ Note: This is only if highly symmetric. Otherwise, we can only get an approximation using numerical solutions with computers ... yes, with COMPUTERS ONLY

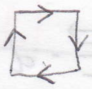
II. $\oint \vec{B} \cdot d\vec{A} = \emptyset$ Poles balance out.

III. $\oint \vec{B} \cdot d\vec{s} = \mu_0 I$ (where $\mu_0 = 4\pi \times 10^{-7} \dfrac{Tm}{A}$)

IV $\oint \vec{E} \cdot d\vec{s} = -N \dfrac{d\Phi_m}{dt}$

COGITATION # 047 Self-Inductance



I_0 in  this direction because it goes from + to − terminals. but, no current when switch is open.

$I = \frac{V}{R}$ when switch is closed, but it takes time to build up to $I = \frac{V}{R}$. Why?

no magnetic field when switch is open.

The charge moves at the speed of light. So why does it take time for the rate of flow to reach $\frac{V}{R}$?

answer: self inductance.

If switch opens, then I decreases, and hence Φ_m decreases $\therefore$ B is same and current direction same. When I increases (when S is closed), then the induced current I_{in} will be going in the opposite direction because Φ_m increases, which means B_f is opposite B_0 — and, using four fingers wrapping in direction of the current with thumb pointing in B_f direction ... eureka (right hand rule version 3)

$$\mathcal{E}_L = -N \frac{d\Phi_m}{dt} = -L \frac{dI}{dt}$$

where $-L \frac{dI}{dt}$ is based on the current change.

L: inductance (similar to capacitance) $L \rightarrow$ Henries, $L \rightarrow \frac{V \cdot s}{A}$

$$\mathcal{E}_L = -N \frac{d\Phi_m}{dt} = -L \frac{dI}{dt}$$

$$L = \frac{\mathcal{E}_L}{-\left(\frac{dI}{dt}\right)} = \frac{-N\frac{d\Phi_m}{dt}}{-\left(\frac{dI}{dt}\right)}$$

$$L = \frac{\mathcal{E}_L}{-\left(\frac{dI}{dt}\right)} = \frac{N\Phi_m}{I}$$

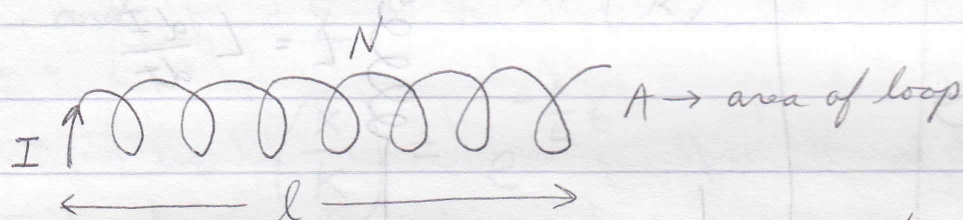
note: $\Phi_m = \oint \vec{B}\cdot d\vec{A}$

remember?

$\tau = NBAI$

and $L = \frac{NBA}{I}$

for a solenoid L



$$L = \frac{N\Phi_m}{I} = \frac{NBA}{I} = \frac{N\mu_0 \frac{N}{l} IA}{I}$$

$$L = N^2 \mu_0 \frac{A}{l}$$

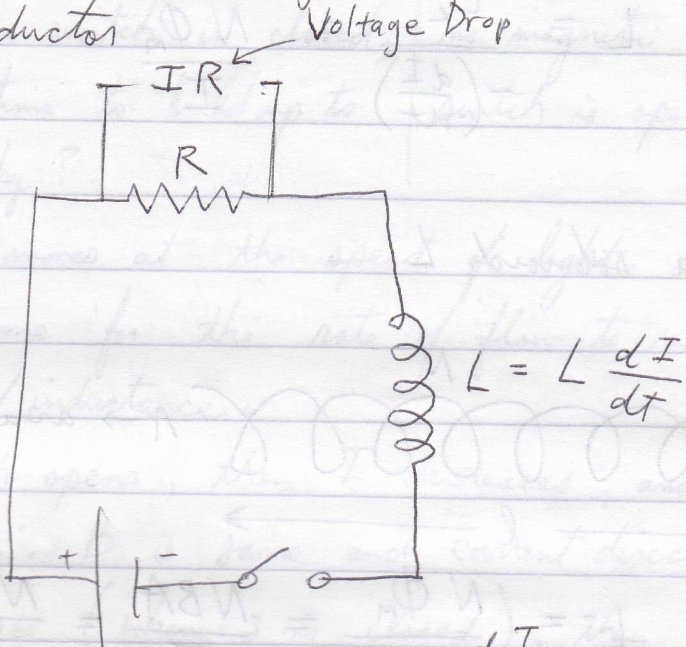
COGITATION #Ø48 RL - circuit

resistor inductor

If L is larger, the circuit is an inductor such as a solenoid or many coils.

—www— resistor (see logbook #59)

—ooooo— inductor



$$\mathcal{E} = IR + L\frac{dI}{dt}$$

eventually, $I = \mathcal{E}/R$

Again, it takes time because of self inductance

SOLVE FOR I: $\mathcal{E} - IR = L\frac{dI}{dt}$

tricky: rewrite $\mathcal{E} - IR$ as $R\left(\frac{\mathcal{E}}{R} - I\right)$. They are the same

now, $R\left(\frac{\mathcal{E}}{R} - I\right) = L\frac{dI}{dt}$

let $x \leftarrow \left(\frac{\mathcal{E}}{R} - I\right)$: now, $Rx = L\frac{dI}{dt}$

let $dx = 0 - dI$

$dx = -dI$; $dI = -dx$

$$\left.\begin{matrix} dx = 0 - dI \\ dx = -dI ;\ dI = -dx \end{matrix}\right\} Rx = L\frac{-dx}{dt}$$

$$Rx = L\frac{-dx}{dt}$$

where $x = \frac{\varepsilon}{R} - I$, $dx = -dI$, $dI = -dx$

$$\int \frac{-R}{L}\,dt = \int \frac{dx}{x}$$

$$-\frac{R}{L}t = \ln x + C$$

let $C = -\ln k$, then $-\frac{R}{L}t = \ln x - \ln k$

and $-\frac{R}{L}t = \ln\left(\frac{x}{k}\right)$

$$\left(\frac{x}{K}\right) = e^{-\frac{R}{L}t}$$

$$x = ke^{-\frac{R}{L}t}$$

$$x = \frac{\varepsilon}{R} - I = ke^{-\frac{Rt}{L}}$$

$$I = \frac{\varepsilon}{R} - ke^{-\frac{Rt}{L}}$$

$e^{-\frac{t}{\frac{L}{R}}}$ || same as $e^{-\frac{Rt}{L}}$

$\frac{L}{R} = \tau$
TIME CONSTANT FOR RL circuit.

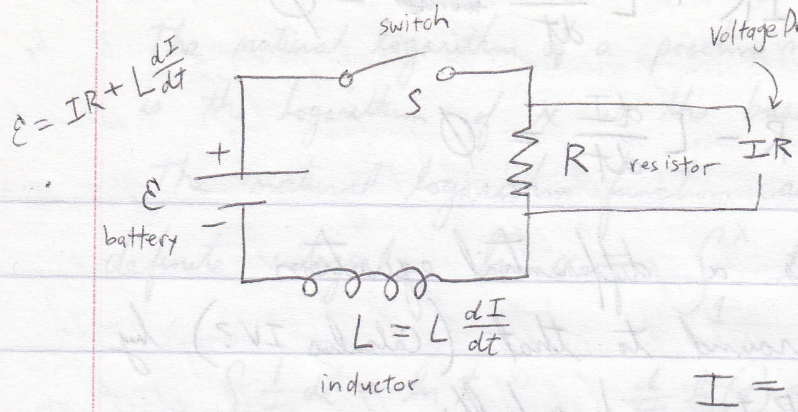
when time $t = \emptyset$, $I = \emptyset$, and $\emptyset = \frac{\varepsilon}{R} - ke^{0}$

$$\emptyset = \frac{\varepsilon}{R} - k \Longrightarrow k = \frac{\varepsilon}{R}$$

$$I = \frac{\varepsilon}{R} - \frac{\varepsilon}{R}e^{-\frac{Rt}{L}} = \frac{\varepsilon}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$$

Current increases with time. $\frac{L}{R}$ is in seconds

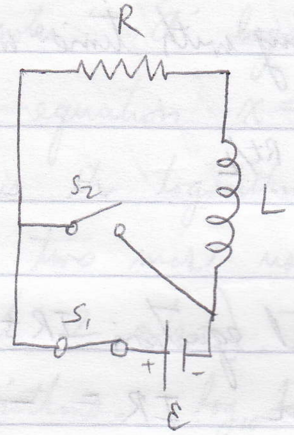
COGITATION # 049 More about R-L circuits



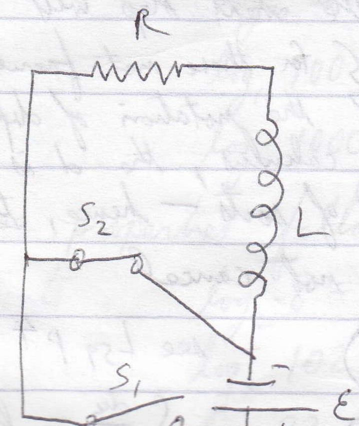
how current increases gradually with time after switch is closed

$$I = \frac{\mathcal{E}}{R}\left(1 - e^{-Rt/L}\right)$$

This equation was derived in cog48p7 (herein)



at time $t = \emptyset$, $I = \frac{\mathcal{E}}{R}$



how current decreases with time:

$$I = \frac{\mathcal{E}}{R} e^{-Rt/L}$$

At time $t = \emptyset$, S_1 is thrown open and S_2 is thrown closed → the circuit then has no battery ($\mathcal{E} = \emptyset$). Applying Kirchhoff's circuit law to the upper loop containing the resistor and the ~~conductor~~ inductor, we obtain $IR + L\frac{dI}{dt} = \emptyset$

$$\mathcal{E} = IR + L\frac{dI}{dt} = \emptyset$$

$$\mathcal{E} - IR - L\frac{dI}{dt} = \emptyset$$

$IR + L\frac{dI}{dt} = \emptyset$ is a differential equation. I will be getting around to that (Calculus IV?) by September 2000 at Rutgers hopefully.

recall the equation for current decreasing with time where we let $\tau = \frac{L}{R}$ and $I_0 = \mathcal{E}/R$

$$I(t) = I_0 e^{-t/\tau} = \frac{\mathcal{E}}{R} e^{Rt/L}$$

Is this a solution to the differential equation $IR + L\frac{dI}{dt} = \emptyset$

$$-IR = L\frac{dI}{dt} \quad \text{or} \quad IR = -L\frac{dI}{dt}$$

whichever one we choose, the next step will look like this:

$$\frac{-R}{L}dt = \frac{dI}{I}$$

for those not familiar with the notation of differential calculus, the d is an operator of sorts — hence, the I's do not cancel

Next, we take the integral of both sides:

$$\int -\frac{R}{L}\,dt = \int \frac{dI}{I}$$

$$-\frac{R}{L}\int dt = \int \frac{1}{I}\,dI$$

$$-\frac{R}{L}t = \ln\frac{I}{A}$$

$$Ae^{-Rt/L} = I$$

see L_{59} p4:

$$\int \frac{du}{u} = \ln(u) + c$$

let $u = I$

then $\int \frac{dI}{I} = \ln(I) + c$

$\ln(I) + C = \ln(I) - \ln(A)$

at $t = \emptyset$, $A = \frac{\mathcal{E}}{R}$ ~~because~~ $e^{0/L} = 1 \longrightarrow I = \frac{\mathcal{E}}{R}e^{-Rt/L}$

see PAGE 17.

COGITATION #050 interlude: Natural Logarithm 11

The natural logarithm of a positive number x (written as $\ln x$) is the logarithm of x is the base e, where $e = 2.71828\ldots$

The natural logarithm function can also be defined by the definite integral $\ln x = \int_1^x t^{-1}\,dt = \int_1^x \frac{1}{t}\,dt$

$\int \frac{1}{t}\,dt = \ln t$; $\frac{d}{dt}\ln(t) = \frac{1}{t}$

side notes: A logarithm is an inverse of an exponential. The equation $x = a^y$ can be written as $y = \log_a x$ "y is the logarithm to the base a of x". The two most useful bases are 10 and e (and I think 2 also). Logarithms to the base 10 are common logarithms. $\log_{10} 1 = 0$ because $10^0 = 1$

$\log_{10} 10 = 1$

$\log_{10} 100 = 2$

$\log_{10} 1000 = 3$

These properties follow directly from the properties of exponents.

$$\left.\begin{aligned} \log_2 8 &= 3 \\ \log_2 1024 &= 10 \\ \log_2 32 &= 5 \end{aligned}\right\}$$

Calculators are in $\log_{10}$ usually
Type in $(\log 32 / \log 2)$
result $\longrightarrow$ 5

So, it follows that $\log_e e = 1 = \ln e$

$\ln$ is the notation for $\log_e$.

$\ln 32 \approx 3.4657\ldots$ less than $\log_2 32$.

$\log_{10} 32 \approx 1.5051\ldots$ The lower the base, the higher the logarithm.

11 This is an intuitive idea: the ~~high~~ lower the exponent, the higher the number being raised to that power has to be.

Physics II has exposed me to how important higher mathematics is! Each math course I take at Rutgers will give me that much more mental power for my computer science courses. I have yet to really get my money's worth out of REA's Problem Solvers Calculus. This will start on December 24th. I will be taking notes in L61.

I will try to branch some of these Physics motivated problems into L61. These notes on logarithms may be repeated in L61, so I will keep it short:

$$y = \ln x$$

$$x = e^y$$

$$\frac{dx}{dx} = e^y \frac{dy}{dx} \Longrightarrow 1 = e^y \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{e^y} = \frac{1}{x}$$

How does this help with cog49?

$$\int \frac{1}{I}\, dI = \ln I + \text{"some constant"}$$

in $-\frac{R}{L}\int dt$, $\int dt = t$ $\therefore$ $-\frac{Rt}{L} = \ln(I) + C$

We make each side of the equation an exponent of e.

$$Ce^{-Rt/L} = I \qquad \text{at } t = 0,\ I = \frac{\varepsilon}{R}$$

I will get back to $\ln(x)$ on December 24th in L61 for some

COGITATION #Ø51 Energy in the Magnetic Field.

recall that for a parallel plate $C = \frac{\epsilon_0 A}{d}$

$$U = \frac{1}{2} \frac{\epsilon_0 A}{d} V_{voltage}^2 \quad \text{because} \quad U = \frac{CV^2}{2}$$

$$U = \frac{1}{2} \frac{\epsilon_0 A V^2}{d} = \frac{1}{2} \frac{\epsilon_0 A V^2 d}{d^2} = \frac{1}{2} \epsilon_0 A d \left(\frac{V}{d}\right)^2$$

$V = Ed$

$\therefore E = \frac{V}{d}$

$\left(\frac{V}{d}\right)^2 = E^2$

$$U = \frac{1}{2} \epsilon_0 A d E^2 = \frac{1}{2} \epsilon_0 V_{volume} E^2$$

This U is the amount of energy the electric field is carrying.

concept: energy density $u \longrightarrow \frac{\text{energy}}{\text{volume}} \longrightarrow \frac{U}{V}$

energy density is proportional to E^2

So $\frac{U}{Ad}$ becomes u $\left[\frac{\text{energy in joules}}{\text{volume}}\right]$

$$u = \frac{1}{2} \epsilon_0 E^2$$

recall that $I\mathcal{E} = I^2 R + LI\frac{dI}{dt}$

Power in Watts ~~by~~ BATTERY — Power dissapated in resistor — power stored in the inductor

$P_s = \mathcal{E} LI \frac{dI}{dt}$ $\therefore$ energy dissapation rate $\rightarrow$ Power

$$\frac{dU}{dt} = P_s = \mathcal{E} LI \frac{dI}{dt} \Longrightarrow U = \frac{LI^2}{2}$$

13

I will repeat that last part:

$$I\mathcal{E} = I^2 R + LI\frac{dI}{dt}$$

Power From Battery = Power Dissapated in Resistor + Power Stored in Inductor

This expression tells us that the rate at which energy is supplied by the battery, $I\mathcal{E}$, equals the sum of the rate at which joule heat is dissapated in the resistor, I^2R, and the rate at which energy is stored in the inductor, $LI\frac{dI}{dt}$.

It is simply an expression of energy conservation.

Let U_B represent the energy stored in the inductor at any time, where subscript B denotes energy stored in the magnetic field of the inductor.

The rate, $\frac{dU_B}{dt}$, at which energy is stored, can be written

$$\boxed{\frac{dU_B}{dt} = LI\frac{dI}{dt}}$$

To find the total energy stored in the inductor, we rewrite as $dU_B = LI\,dI$ and integrate:

$$U_B = \int_0^{U_B} dU_B = \int_0^I LI\,dI = L\int_0^I I\,dI$$

$\frac{x^{n+1}}{n+1}$

$$U_B = L\frac{I^2}{2}\Big|_0^I = \frac{LI^2}{2}$$

(similar to $U = \frac{Q^2}{2C}$)

Consider the solenoid whose inductance is given by:

$$L = \mu_0 n^2 A l$$

the magnetic field of a solenoid is given by:

$$B = \mu_0 n I \quad \therefore \quad I = \left(\frac{B}{\mu_0 n}\right)$$

$$U_B = \frac{1}{2} L I^2 = \frac{1}{2} \mu_0 n^2 A l \left(\frac{B}{\mu_0 n}\right)^2 = \frac{B^2}{2\mu_0}(Al)$$

$$u_B = \frac{U_B}{Al} = \frac{B^2}{2\mu_0}$$ MAGNETIC ENERGY DENSITY

Another way to derive this equation is just brute force:

$$L = \frac{N^2 \mu_0 A}{l} \; ; \quad U_B = \frac{LI^2}{2} = \frac{N^2 \mu_0 A I^2}{2l}$$

$$\frac{N}{l} = n$$

$$U_B = \frac{N^2 \mu_0 A I^2}{2l} \cdot \frac{l\mu_0}{l\mu_0} = \frac{N^2 \mu_0^2 A I^2 l}{2l^2 \mu_0} = \frac{n^2 \mu_0^2 I^2 A l}{2\mu_0}$$

$$U_B = \frac{(n\mu_0 I)^2 A l}{2\mu_0} = \frac{B^2 A l}{2\mu_0}$$

$B = n\mu_0 I$ for solenoid

$$\therefore u_B = \frac{U_B}{Al} = \frac{B^2}{2\mu_0}$$

342 09:30 hrs I will miss "working" with my father. 17

The job we were to do, with included unloading a walk-in freezer, was postponed so we are at another job. I only had to help run the copper pipe for the ice machine. Now I am in the truck thinking about logarithms, which brings us to an afterthought to cog 50 p 11.

COGITATION #Ø52 Some Useful Properties of Algorithms

(may be repeated in L_{61}. All cogitations may be repeated as needed)

$$\ln_e x = (2.3\emptyset 2585) \log_{10} x$$

where $e \approx 2.71828\ldots$

$$\log(ab) = \log a + \log b$$

$$\log(a/b) = \log a - \log b$$

$$\log(a^n) = n \log a$$

$\ln e = 1$ just as $\log_{10} 10 = 1$ or $\log_x x = 1$ because $x^1 = x$

$\ln e^a = a$ just as $\log_{10} 10^a = a$ by definition

$$\ln\left(\frac{1}{a}\right) = -\ln a$$

So, what is a cogitation that experiments with an unclear concept? a thought experiment? From $IR + L\frac{dI}{dt} = \emptyset$ at time $t = \emptyset$;

$$IR = -L\frac{dI}{dt} \Rightarrow \frac{R}{-L}dt = \frac{dI}{I} \Rightarrow \frac{-R}{L}\int dt = \int \frac{dI}{I}$$

$\frac{-Rt}{L} = \ln(I) + C = \frac{\ln(I)}{(I_0)}$ where the integrating constant C is taken to be $-\ln(I_0)$. Taking the antilog of this result gives $I = I_0 e^{-Rt/L}$. Since at $t = \emptyset$, $I = \emptyset$, we note ~~that~~ $\cancel{I_0 = \frac{\varepsilon}{R}, \text{ hence } \frac{\varepsilon}{R} \; I = \frac{\varepsilon}{R} e^{-Rt/L} \Rightarrow I = \frac{\varepsilon}{R}\left(1 - e^{-Rt/L}\right)}$

F1 In order to solve Physics problems, one requires the formula; but one does not require Calculus. Calculus becomes necessary when deriving the formula itself! This is why I hold Pure Mathematics superior to Applied Science.

I dont "have to" understand the mathematical derivation of the formula for the nature of how a current decreases ~~with~~ over time. I want to understand. That is why I have basically rewritten cog 50 p. 11. Now I write it with understanding. Now I grok.

$X = a^Y$ number a is the base

The logarithm of x with respect to the base a is equal to the exponent to which the base number must be raised in order to satisfy the expression $x = a^y$: $y = \log_a x$

Conversely, the antilogarithm of y is the number

$X = \text{antilog}_a Y \quad (X = a^Y)$

$a^Y = \text{antilog}_a Y \quad \text{antilog}_{10} 2 = 100$

$\text{antilog}_e \frac{-Rt}{L} = e^{-Rt/L} \quad \log_{10} 100 = 2$

$\ln(e^{-Rt/L}) = -\frac{Rt}{L}$ Is this clear?

so when we get to $-\frac{R}{L}\int dt = \int \frac{dI}{I}$ in cog 50 p 11,

$-\frac{Rt}{L} = \ln(I) + C$; we let $C = -\ln(I_0)$

$-\frac{Rt}{L} = \ln(I) - \ln(I_0) = \ln\left(\frac{I}{I_0}\right)$ [Focus in on this]

$$\frac{-Rt}{L} = \ln(I) + C = \ln(I) - \ln(I_0) = \ln\left(\frac{I}{I_0}\right)$$

because we set C equal to $-\ln(I_0)$.

C is a constant. It can be any value. We choose a value that will help us simplify the equation.

$$\frac{-Rt}{L} = \ln\left(\frac{I}{I_0}\right)$$

Here is where we ~~imple~~ exploit the antilogarithmic function I will write out in long for for clarity:

$$\text{antilog}_e\left(\frac{-Rt}{L}\right) = \text{antilog}_e\left(\ln\frac{I}{I_0}\right)$$

The left side is easy enough, but by definition $\text{antilog}_e\left(\ln\frac{I}{I_0}\right)$ would yield

$$e^{\ln\left(\frac{I}{I_0}\right)}$$

This is like $10^{\log_{10} X}$. Look at $\log_{10} X$.

$\log_{10} X$ asks for the exponent needed in $10^Y = X$. This "y" will be the exponent for 10^Y, which will yield x!!!

$b^{\log_b X} = X$! $e^{\ln X} = X$.

$\therefore e^{\ln\left(\frac{I}{I_0}\right)} = \frac{I}{I_0}$!

$$e^{-\frac{Rt}{L}} = \frac{I}{I_0} \Rightarrow e^{\frac{-Rt}{L}} = \frac{I}{I_0}$$

$$I = I_0 e^{\frac{-Rt}{L}}$$

~~at $t=0$ $I_0 = \frac{\mathcal{E}}{R}$ and $I = 0$~~

$$I(t) = \frac{\mathcal{E}}{R} e^{\frac{-Rt}{L}}$$

where $I_0 = \frac{\mathcal{E}}{R}$

COGITATION #Ø53 : Alternating Current Circuits

battery → Direct Current source
~~d~~ converts chemical energy

generator (socket in wall) → AC circuit

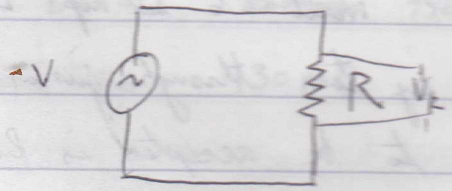
In 1 second, 60 cycle changes occur. (60 Hz)
AC source is a generator. It converts mechanical energy.

$$\Phi_M = BA\cos\theta$$

$$V = V_{max}\sin\omega t$$

$$\omega = 2\pi f \qquad f = 60\text{ Hz} \quad (\text{frequency } f)$$

RESISTORS IN AC CIRCUIT



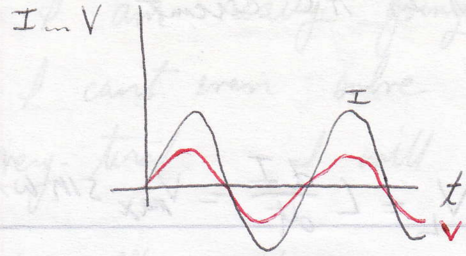
$$I = \frac{V}{R} = \frac{V_{MAX}}{R}\sin\omega t$$

$$I_{MAX} = \frac{V_{MAX}}{R}$$

$$I = I_{MAX}\sin\omega t$$

$$V = V_R = V_{MAX}\sin\omega t$$

$$V_R = I_{MAX}R\sin\omega t$$



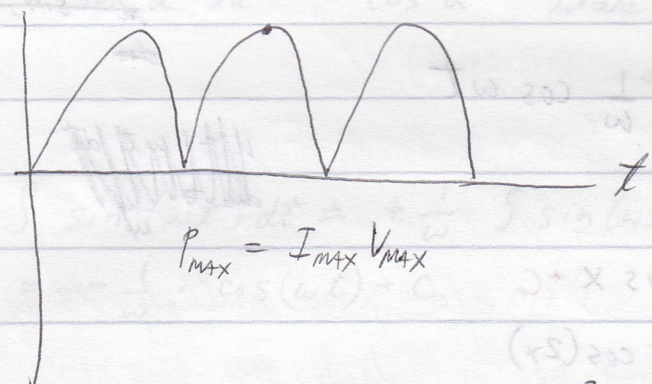
I and V are in phase. They reach their max and min at the same time.

$$\bar{I} = \emptyset, \quad \bar{V} = \emptyset, \quad \bar{P} = \frac{1}{2} P_{MAX}$$

where $P_{MAX} = I_{MAX} V_{MAX}$

$$\bar{P} = \frac{1}{2} I_{MAX} V_{MAX}$$

$$\bar{P} = \frac{V_{MAX}^2}{R} \qquad \bar{P} = \frac{1}{2} I_{MAX}^2 R$$



$$\bar{P} = I_{rms} V_{rms} = \frac{V_{rms}^2}{R}$$

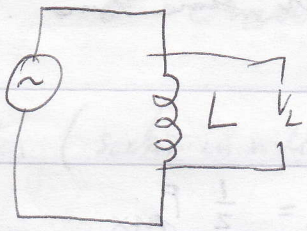
Root Mean Square: $I_{rms}^2 = \frac{1}{2} I_{MAX}^2$

$$I_{rms} = \emptyset.7\emptyset7 \, I_{max}$$

$$V_{rms} = \emptyset.7\emptyset7 \, V_{max}$$

On equipment, we are given root mean square

COGITATION #054: Inductors in AC circuit



$$V = V_L = L\frac{dI}{dt} = V_{max}\sin\omega t$$

$$\frac{L}{V_{MAX}}\, dI = \sin\omega t\, dt$$

$$\int \frac{L}{V_{MAX}}\, dI = \int \sin\omega t\, dt$$

$$\frac{L}{V_{MAX}} \int dI = \int \sin\omega t\, dt$$

$$\frac{LI}{V_{MAX}} = -\frac{1}{\omega}\cos\omega t$$

$$\int \sin x \overset{dx}{=} -\cos x + C$$

$$\int \sin(2x) \overset{dx}{=} -\frac{1}{2}\cos(2x)$$

how so? $\int \sin(\omega t)\,dt \rightarrow \int \sin x\, \cancel{dx} = -\cos x + C$

why? $u = x$ $du = dx = 1$ ∴ $\int \sin x\, dx = -\cos x\,(1)$

so for $\int \sin(\omega t)\,dt$: $u = \omega t$ $du = \omega\, dt$ $dt = \frac{du}{\omega}$

$\int \sin u \frac{du}{\omega} = -\cos u$

$\int \sin(\omega t)\,dt = -\frac{1}{\omega}\cos\omega t$ $\frac{d(x^5)}{dx} = 5x^4$

$\frac{1}{\omega}$ must be dt. ~~The derivative of~~ ωt ~~is~~ ω $\int 5x^4 = \frac{5x^5}{5}$

just as $\frac{d(5t)}{dt} = 5$; ∴ $\frac{d(\omega t)}{dt} = \omega$ ∴ $dt = \frac{1}{\omega}$

I am really going to have to review my calculus. I can't even solve $\int \sin(\omega t)\,dt$. I am nodding off, very tired. I will have to devote all day tomorrow to Physics ch 33 parts 1, 2, 3 as well as Java or 12. I will not get much done tonight as I am fading in and out of consciousness. Dad and I worked hard these past 2 days.

Just how much work I need to put into a Calculus review can be seen here.

$$\int \sin(\omega t)\,dt$$

$$\int \sin u \, du = -\cos u \quad \text{where } u = \omega t$$

$$du = \omega\, dt$$

$$dt = \frac{du}{\omega}$$

$$\int \sin(\omega t)\,dt = +\frac{1}{\omega}\int \sin(\omega t)\cdot dt$$

$$= -\frac{1}{\omega}\cdot \cos(\omega t) + C$$

$$\frac{LI}{V_{MAX}} = -\frac{1}{\omega}\cos \omega t$$

$$I = -\frac{V_{MAX}}{L\omega}\cos \omega t$$

$$\boxed{\cos(\omega t) = \sin(\omega t - \tfrac{\pi}{2})}$$

$$I = \frac{V_{MAX}}{L\omega}\sin(\omega t - \tfrac{\pi}{2})$$

$X_L = \omega L$ (ohms → Ω) Inductance reactance

$$I = \frac{V_{MAX}}{X_L}\sin(\omega t - \tfrac{\pi}{2}) \;;\quad I_{rms} = \frac{V_{rms}}{X_L} \;;\quad I_{MAX} = \frac{V_{MAX}}{X_L}$$

Am I that confused with Integration?

$\int \cos 2x \, dx$

Consider $\int \cos u \, du = \sin u + C$

Let $u = 2x$ then $du = 2\,dx \longrightarrow dx = \frac{du}{2} = \frac{1}{2} du$

To obtain $\int \cos u \, du$, we multiply dx by 2 under the integral sign and by $\frac{1}{2}$ outside the sign.

We obtain $\frac{1}{2} \int \cos 2x \cdot 2\,dx$

$\int \cos 2x \, dx = \frac{1}{2} \sin 2x + C$

Check: $\frac{d\left(\frac{1}{2}\sin 2x + C\right)}{dx} = \frac{1}{2}(\cos 2x)2 = \cos 2x$

Again: $\int \sin(\omega t)\,dt$

Consider $\int \sin u \, du = -\cos u + C$

Let $u = \omega t$ then $du = \omega\,dt \longrightarrow dt = \frac{du}{\omega} = \frac{1}{\omega} du$

To obtain ~~was~~ $\int \sin u \, du$, we multiply dt by ω under the integral sign and by $\frac{1}{\omega}$ outside the sign

We obtain $\frac{1}{\omega} \int \sin(\omega t)\,\omega\,dt = -\frac{1}{\omega}\cos(\omega t) + C$

I will go over fundamental integration in the start of L61 while watching Mad TV and SNL.

Perhaps tomorrow I will be able to continue with Physics Ch 33 problems with a deeper level of understanding.

1999 346 Su 12 December 01:15 hrs I am very tired

In spanish: "I am very hungry" → Yo tango hombre.

I am ready for Calculus, or at least - a calculus review. I know that Multivariable Calculus at Rutgers will be the most challenging math class I have ever taken. I am prepared for a B.

I return to the formula in cog 54:

$$V = V_L = L\frac{dI}{dt} = V_{MAX}\sin(\omega t)$$

$$\frac{L}{V_{MAX}}\,dI = \sin(\omega t)\,dt$$

$$\int \frac{L}{V_{MAX}}\,dI = \frac{L}{V_{MAX}}\int dI = \underbrace{\int \sin(\omega t)\,dt}_{\int \sin u\,du = -\cos u + C}$$

$$\frac{LI}{V_{MAX}} = \frac{1}{\omega}\int \underbrace{\sin(\omega t)}_{\sin u}\,\underbrace{\omega\,dt}_{du} \qquad \text{let } u = \omega t$$

$$du = \omega\,dt$$

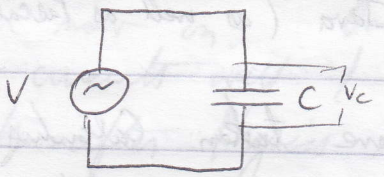
$$\frac{LI}{V_{MAX}} = -\frac{1}{\omega}\cos(\omega t) + C \qquad dt = \frac{1}{\omega}\,du$$

$$I = \frac{-V_{MAX}}{L\omega}\cos(\omega t)$$

$$I = \frac{V_{MAX}}{L\omega}\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$\frac{V_{MAX}}{L\omega} \longrightarrow \frac{V_{MAX}}{X_L} \quad \text{where } X_L = L\omega$$

COGITATION #Ø55 CAPACITORS IN AC CIRCUIT



$$C = \frac{q}{V}$$

$$V = \frac{q}{C}$$

$$V = V_C = \frac{q}{C} = V_{MAX}\ \sin(\omega t)$$

$$Q = CV_{MAX}\ \sin(\omega t) \qquad i = \frac{dQ}{dt}$$

$$i = \frac{dQ}{dt} = \omega C V_{MAX}\cos(\omega t)$$

$$i = \omega C V_{MAX}\ \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$V_C = V_{MAX}\ \sin(\omega t)$$

$$\frac{q}{C} = V_{MAX}\ \sin(\omega t)$$; take derivative of each side

currents in amps

$$\frac{1}{C}\frac{dq}{dt} = \omega V_{MAX}\cos(\omega t)$$

$$\frac{1}{C} I = \omega V_{MAX}\left(\sin(\omega t) + \frac{\pi}{2}\right)$$

$$I = C\omega V_{MAX}\left(\sin(\omega t) + \frac{\pi}{2}\right)$$

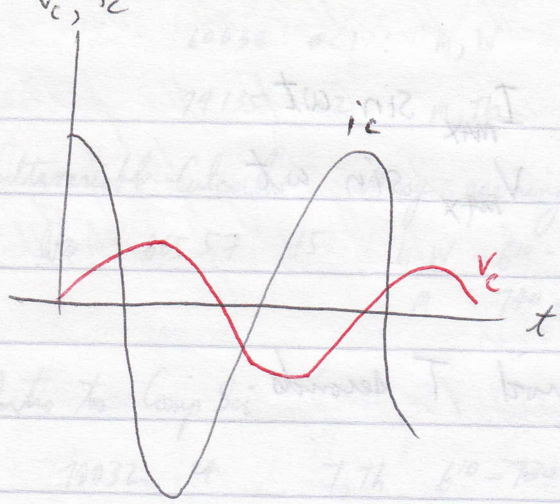
capacitance reactance

$$X_C = \frac{1}{C\omega}\ \Omega$$

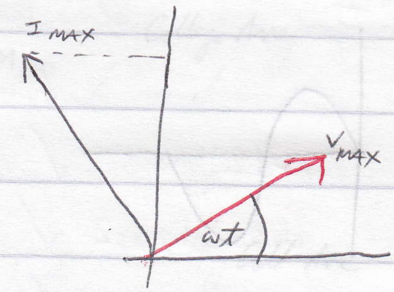
$$I = \frac{V_{MAX}}{\frac{1}{C\omega}}\left(\sin(\omega t) + \frac{\pi}{2}\right)$$

$$I = \frac{V_{MAX}}{X_C}\left(\sin(\omega t) + \frac{\pi}{2}\right)$$

$$V_c = V_{MAX} \sin(\omega t) = I_{MAX} X_c \sin(\omega t)$$



voltage lacks current by 90° for capacitor



346 14:25 hrs I have only ten days to digest these concepts while learning more concepts that will also be covered on the final exam December 23rd. I will do what I can of Ch 33 problems, but then I want to attack the Java assignments.

346 17:00 hrs From internet h.w.sweethaven.com/acee/forms/ The Sinusoidal AC Waveform frm0101.htm

represented by sin function $y = r \sin\theta$

where y = instantanious amplitude

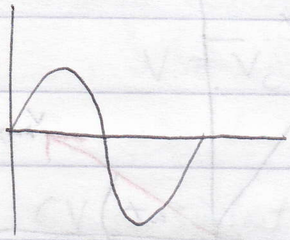
r = maximum amplitude

θ = horizontal displacement

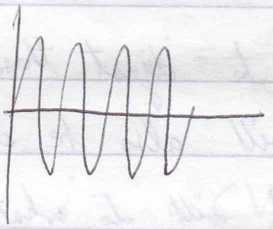
horizontal axis represents angular displacement.

instantaneous current $i = I_{MAX} \sin \omega t$

instantaneous voltage $v = V_{MAX} \sin \omega t$



period T seconds



frequency f Hz $= f\ s^{-1}$

$$f = \frac{1}{T}$$

period of frequency $f = \frac{1}{T}$

frequency of period $T = \frac{1}{f}$

348 17:00 hrs I will be working tomorrow — at least I will have earned some cash; I have much Physics to go over this and tomorrow evening. Before getting into cogitations about ch 33, ch 35 and ch 36 Physics II problems, I will make a note of bubblesort() used in crhh1. 47

COGITATION #Ø56 sorting strings that were array elements of objects ~~the~~ (an array of objects).

```
public Student[] bubblesort (Student[] list, int elements) {
    // this gets called as:
    // Student[] sortedStudents = new Student[students];
    // sortedStudents = bubblesort(s, students);
    // where students is a counter and s is an array of Student objects
    for (int pass = 1; pass < elements; pass ++) {   // passes
        for (int i = Ø; i < elements - 1; i++) {   // one pass
            String s1 = list[i].data[Students.SURNAME];
            String s2 = list[i+1].data[Students.SURNAME];
            if (s1.compareTo(s2) > Ø)   // one comparison
            {
                swap(list, i, i+1);   // one swap
            }
        }
    return list;
}
```

```
public void swap(Student[] list, int first, int second) {
    Student hold = new Student(list[first]);
    list[first] = new Student(list[second]);
    list[second] = new Student(hold);
}
```

This is elegant code, considering that list is an array of objects that contain arrays themselves. Here is the copy constructor that takes its own "type/class" object as an argument; Where static final int SURNAME=0, FORENAME=1, CLASS=2, COLLEGE=3, MAJOR=4;

```
public Student(Student student) {
  { data[[Student.SURNAME] = student.data[student.SURNA
    ;
    ;
  }
```

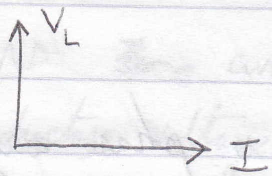
or simply

```
    for (int i = 0; i < ELEMENTS; i++) {
        data[i] = student.data[i]
    }
```

Anyway, I have two evening to work on Physics II problems. Over the weekend I will be busy with Java. Next week at this time I will be preparing for exams, finish java crh2 and crhh2, and winding down waiting to hear from

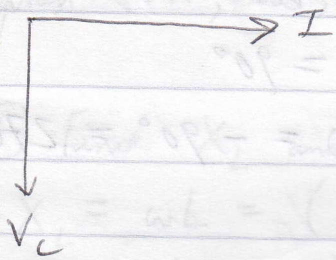
COGITATION #057 AC Circuits Review

$I_R = \frac{V_R}{R}$ for resistors



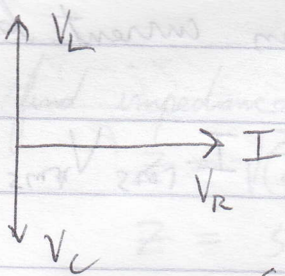
$I_L = \frac{V_L}{X_L}$ for inductors

where $X_L = \omega L = 2\pi f L$

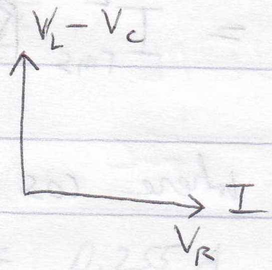


$I_C = \frac{V_C}{X_C}$ for capacitors

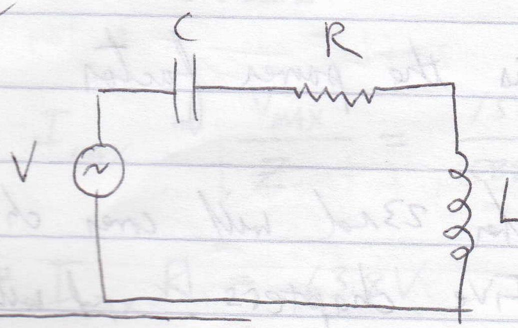
where $X_C = \frac{1}{\omega C}$



$I = \frac{V}{Z}$ impedance

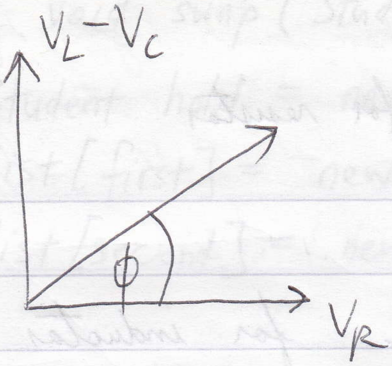


RLC in series



$$V = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{I^2R^2 + (IX_L - IX_C)^2}$$

$$V = I\sqrt{R^2 + (X_L - X_C)^2} \; ; \; \frac{V}{I} = Z = \sqrt{R^2 + (X_L - X_C)^2}$$



$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R}$$

where ϕ is phase angle

if only resistor, phase angle $\phi = 0°$
if only inductor, phase angle $\phi = 90°$
if only capacitor, phase angle $\phi = -90° = 270°$

If phase angle is negative, then current leads.

$$P = I_{rms}^2 R = I_{rms} (V_{rms})_R = I_{rms} V_{rms} \cos\phi$$

where $\cos\phi$ is the power factor

The exam on December 23rd will cover ch 31, 32, 33, 35, and 36: Five chapters! I will write up a cogitation on an AC circuit example, write up a cogitation on Light, and then start homework. I will begin SS on Friday night.

Analyzing a Series RLC Circuit

$R = 250\,\Omega$, $L = 0.6\,H$, $C = 3.5\,\mu F = 3.5 \times 10^{-6}\,F$,
$\omega = 377\,s^{-1}$, and $V_{MAX} = 150\,V$

TO ANALYZE means to FIND EVERYTHING :
FIND ~~the~~ current I , voltage V, phase angle ϕ , inductors voltage V_L , capacitors voltage V_c , resistors voltage V_R , inductive reactance X_L , capacitive reactance X_c , and impedance Z .
given : R , ω , L , C , V_{MAX} .

① Find X_L and X_c

$$X_L = \omega L = (377\,s^{-1})(0.6\,H) = 226\,\Omega$$
$$X_C = 1/(\omega C) = 1/(377\,s^{-1})(3.5\times10^{-6}\,F) = 758\,\Omega$$

② find impedance $Z = \sqrt{R^2\,(X_L - X_C)^2}$

$$Z = \sqrt{(250\,\Omega)^2 + (226-758)^2\,\Omega^2}$$
$$Z = 588\,\Omega$$

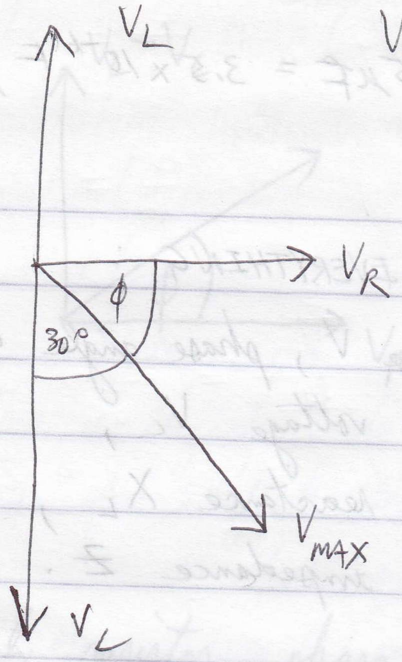
③ find $I_{MAX} = \dfrac{V_{MAX}}{Z} = \dfrac{150V}{588\,\Omega} = 0.255\,A$

④ $V_R = I_{MAX} R = 63.8V$
$V_L = I_{MAX} X_L = 57.6V$
$V_c = I_{MAX} X_c = 193\,V$

⑤ $\phi = \sin^{-1}\left(\dfrac{X_L - X_C}{R}\right)$

$\cancel{\phi = \dfrac{V_L - V_C}{V_R}}$

$\phi = -2.12 \longrightarrow$ current leads



$V_{MAX} = 120V$

$f = 60 Hz$

$R = 800 \Omega$

$C = 4 \mu F = 4 \times 10^{-6} F$

The value of L such that the voltage across the capacitor is out of phase with the applied voltage by 30°, with V_{MAX} leading V_C.

The phase angle ϕ is 60°. This is because the phasors representing I_{MAX} and V_R are in the same direction (they are in phase)

$$X_L = X_C + R \tan \phi$$

why? Because $\tan \phi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R}$

$X_L = 2\pi f L$

$X_C = \frac{1}{2\pi f C}$

hence $2\pi f L = \frac{1}{2\pi f C} + R \tan \phi$

$$L = \frac{1}{2\pi f}\left[\frac{1}{2\pi f C} + R \tan \phi\right]$$

$L = 5.44 \ H$

note about rms

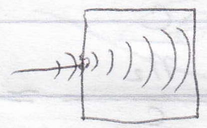
$I_{rms} = 0.707 \ I_{MAX}$

$V_{rms} = 0.707 \ V_{MAX}$

$\overline{P} = I_{rms} V_{rms} = V_{rms}^2 / R$

COGITATION #058 The Nature of Light and the Laws of Geometric Optics

dual nature of light : particle and wave



speed of light $c = 3 \times 10^{8}\ m/s$ in a vacuum

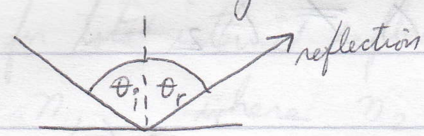
depends on material : $\frac{c}{v} = \cancel{n}\ n$ index of refraction

v is the speed of light for a certain material

$n \geq 1$

Ray approximation in geometric optics

reflection

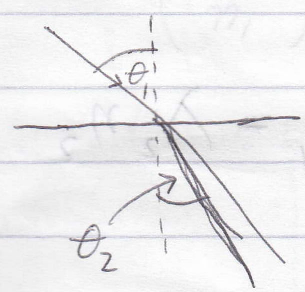


θ_i incident angle

θ_r reflection

$\theta_i = \theta_r$

refraction



~~the~~ light that is absorbed bends

$\left.\begin{matrix}\theta_1 \\ \theta_2\end{matrix}\right\}$ perpendicular to surface

$\theta_1 \neq \theta_2$

relationship $\frac{V_2}{V_1} = \frac{\sin\theta_2}{\sin\theta_1} = \text{constant}$

$$n_1 = \frac{c}{v_1} \qquad n_2 = \frac{c}{v_2}$$

$$v_1 = \frac{c}{n_1} \qquad V_2 = \frac{c}{n_2}$$

$$\therefore \quad \frac{V_2}{V_1} = \frac{\frac{c}{n_2}}{\frac{c}{n_1}} = \frac{n_1}{n_2} = \frac{\sin\theta_2}{\sin\theta_1}$$

$$n_2 \sin\theta_2 = n_1 \sin\theta_1$$

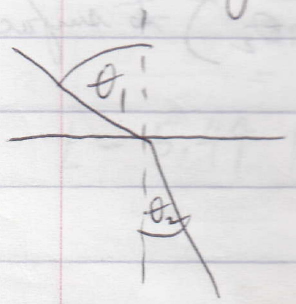
$$v = f\lambda \qquad f_1 = f_2 = f$$

$$v_1 = f\lambda_1 \quad \text{and} \quad v_2 = f\lambda_2$$

$$v_1 \neq v_2 \quad \text{and} \quad \lambda_1 \neq \lambda_2$$

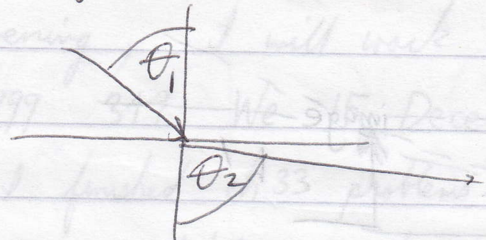
$$n = \frac{c}{v} \quad \therefore \quad \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1}$$

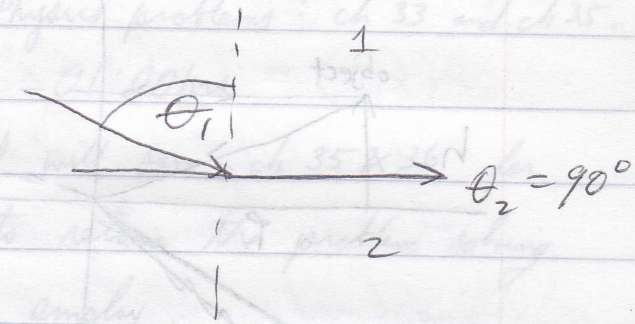
which gives $\lambda_1 n_1 = \lambda_2 n_2$



Total Internal Reflection $n_1 \sin\theta_1 = n_2 \sin\theta_2$

from greater index to less index $n_1 > n_2$





critical angle is when θ_1 causes θ_2 to be $90°$

$$n_1 \sin\theta_c = n_2 \sin 90°$$

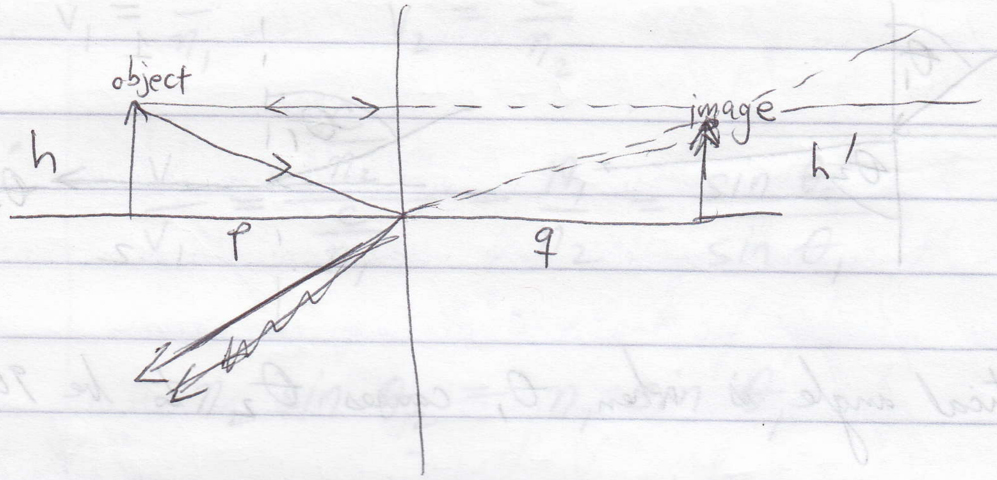
$$\sin\theta_c = \frac{n^2}{n_1} \leq 1$$

In order for total internal reflection to take place, $n_2 \leq n_1$, where n_2 is the second material.

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

COGITATION #059 Geometric Optics

images formed by flat mirrors



p object distance ; q image distance

h object height ; h' image height

lateral magnification $M = \frac{h'}{h}$

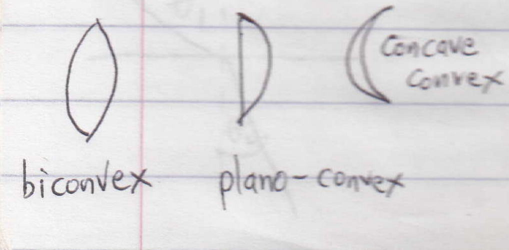
if the mirror is flat : $p = q$

$h = h'$

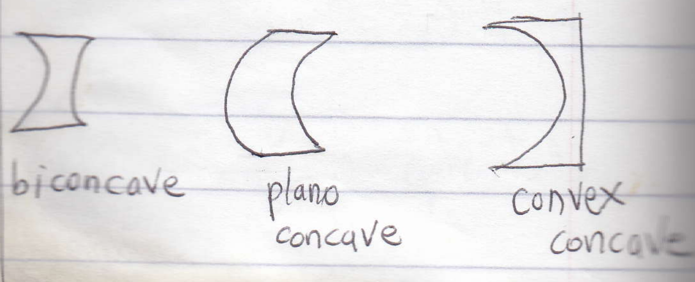
$M = 1$

CONVERGING LENS	THIN LENSES	DIVERGING LENS

convex lens



concave lens



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Lapses Into Melancholia

1999 351 Fr 17 December 23:00 hrs

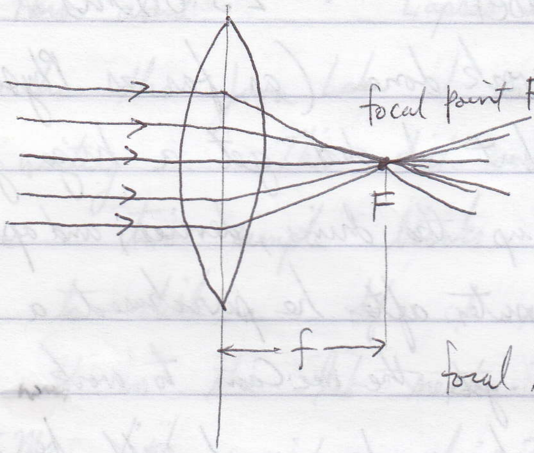
I did not get any work done (as far as Physics, Java, and OST go), but I did get a huge job accomplished: I set up the drives, devices, and operating system on my nephews computer after he purchased a new mother board. We even got the weeCam to work. Now he just has to set up his modem. I will be tidying up my desk and working on Physics. Tomorrow I will start working on the Java exam. I will try to finish it by tomorrow (23:00 hrs).

I got some Yuletide shopping out of the way: Dad, Joe, Tami, and Joey. I only have Janine, Ashley, and Mom left. Now I can begin cogitations on ch 36. These will be the last Physics oriented cogs. The forthcoming Calculus cogs will be placed strategically in L62. I am so anxious to hear from Rutgers, the State University of New Jersey. If I do not start there on January 18th, I do not know how I will react. Melancholia is an understatement. I will be depressed.

COGITATION #060 Lenses

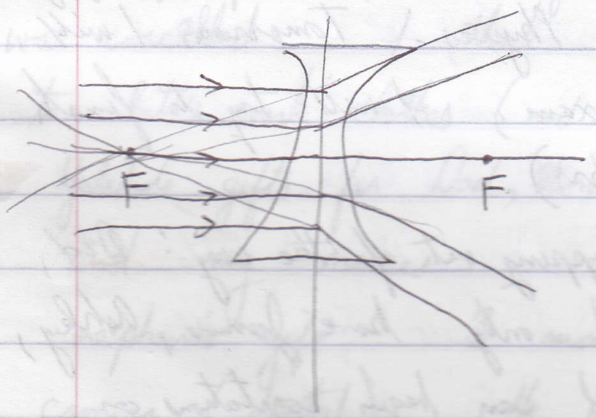
Every lens, whether converging (convex) or diverging (concave), has two equal focal lengths f.

Through a convex lens, parallel light will converge!

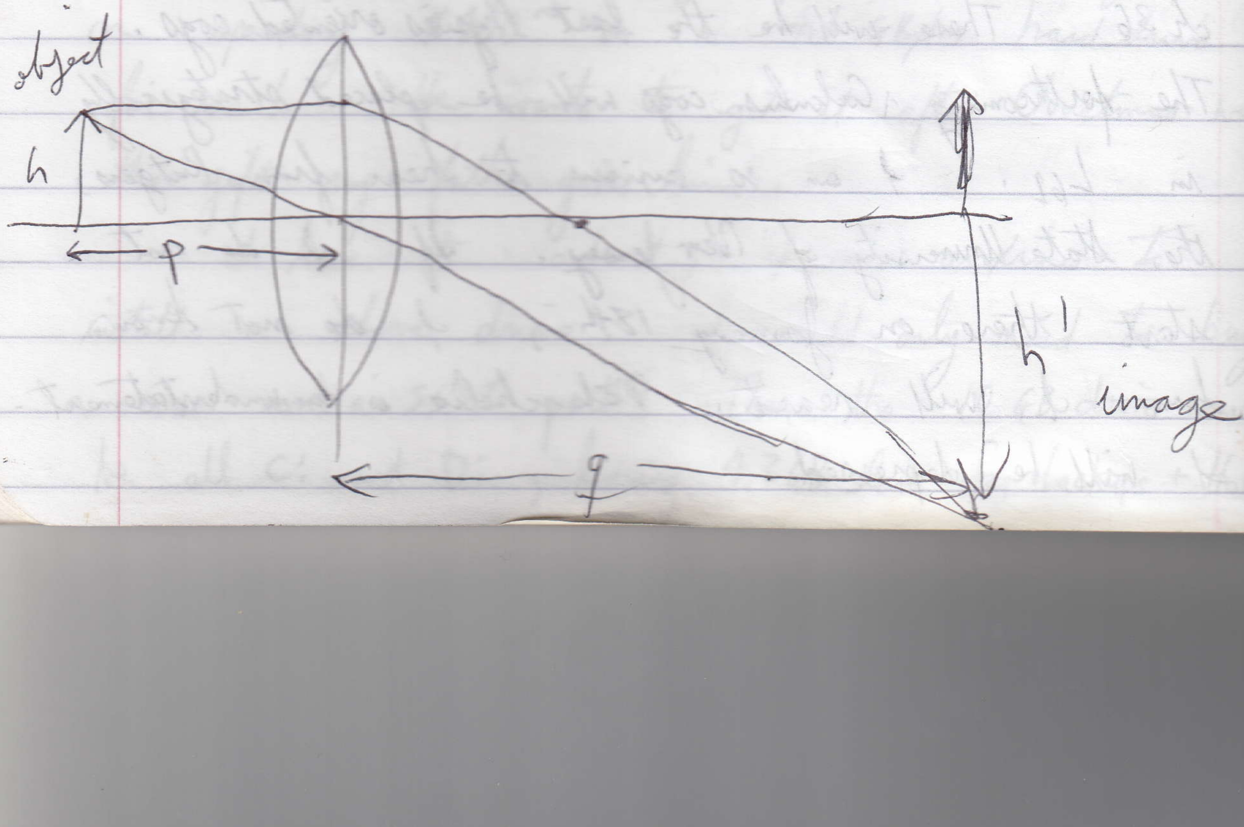


for converging
$+f$

focal length : distance to FOCAL POINT



for diverging
$-f$



$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$ where each can be either + or − as follows:

p+ object in front of lens
p− object in back of lens

q + image in back of lens
q − image in front of lens

f + converging lens (convex)
f − diverging lens (concave)

$$M = \frac{h'}{h} = \frac{-q}{p}$$

h + upright
h − upside down

M + image upright, virtual
M − image upside down, real

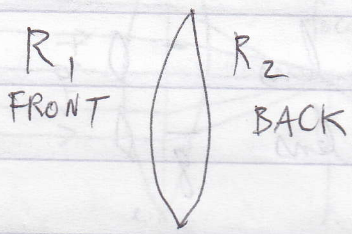
When $M = 1$, $p = q$ ∴ $\frac{1}{p} + \frac{1}{p} = \frac{1}{f}$; $f = 2p$

index of refraction $\frac{c}{v}$

$$\frac{1}{f} = \left(\frac{n_{lens}}{n_{medium}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

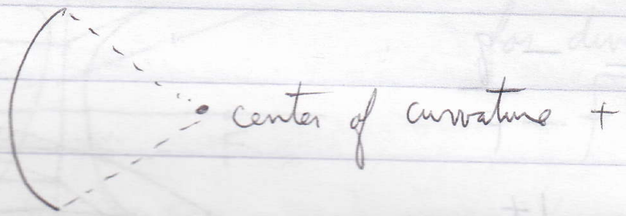
index of refraction for medium

[LENS MAKER'S EQUATION]

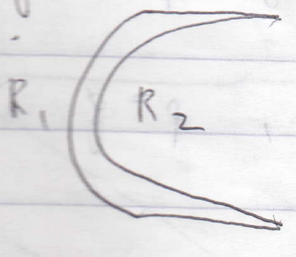


R + : center of curvature ~~in front of~~ ~~behind lens~~ BACK ~~FRONT~~

R − : center of curvature ~~behind lens~~ FRONT



note that for radii of lenses, $R_1 > R_2$ when R_1 is "smoother".



351 23:45 hrs Am I ready to finish the last 7 physics problems for the semester? I think so. Then, after I finish the Java exam, I can create the SS. I will remain confident that my natural, methodical ~~m~~ inner processes will be able to guide me.

I think my nephew really does appreciate the work I put into his machine. I am part of the ghost in his machine; and here I use ghost in my favorite context: "part of the ghost", part of the "invisible processes" of the machine.

1999 352 Sa 18 December 01:00 hrs

My nephew must be in a very "dark place". Imagine how I feel after devoting the last 24 hours to his machine. Now he can't get his ~~non~~ plug and play operating system to detect his non-plug n play modem. My 56K external modem is dead, Otherwise he may have had my internal Flex modem.

There is only one option, but even that is unlikely. My old 33kbps is in the same boat... If he has to buy a modem, he won't be buying that hard drive for me. If he doesn't buy a modem, he will be wasting the internet access I purchased for him.

It is a depressing situation. I will tell him to just purchase a modem (external) if he cannot get his machine to acknowledge the one he's got. What about system.ini? What about DOSPORTS.EXE?

I will make a copy of DOSPORTS.EXE for him. Also, his BIOS is acting up: checkerror sum?

checksum error: The BIOS has failed.

Make sure BIOS IC's are inserted properly and completely into their sockets. Make sure the orientation of pin 1 is correct. Reprogram the flash BIOS?

With all this energy being expended upon my nephew's computer, I have wasted valuable time that could have been utilized on my studies. I dont care. I will get by. I am sure I will preserve 3 A's out of 3 courses.

Why should my nephew be able to access the ports for his modem? Uhgh.. Fuck it. It will all work out. I dont think I will be getting the hard drive though. His having a functional modem is much more important than my having another 4GBytes of memory! Absolutely.

Let him work on it for a week, but then he should just buy a new modem - external. Shall I work on Physics problems or just kill myself? Give Rutgers another 6 days... 5 days... 4 days

I will also call Rutgers and DVR on Monday if (when) I dont receive my acceptance letter. This is getting to be an emergency. After I finish the Physics study sheet, I will set up the computer for my mom.. Will I need a power cord or just an extension?

1999 353 Su 20:00 hrs

The internal cache failure in my nephews pc was a sign that the CPU needed to be replaced. It must have burned up when it overheated. I lent him $100 for a 450 MHz AMD K-6 chip, but we exchanged chips. We put my 400 MHz chip in his; and I put the 450 MHz in mine.

Bottom line: I will not be getting a second hard drive until mid-February. He will pay me the $100 after Christmas. His system is running smoothly now.

To change settings from 400 MHz to 450 MHz, I went to jumper J1 and changed from top to bottom:

	1-2	3-4	5-6
TOP (400 MHz) →	SHORT	OPEN	SHORT
BOTTOM (450 MHz)	SHORT	SHORT	SHORT

Now I shrunk E:DATA from 1G to 670 MB, and I created an H:SWAP 320 MB = 2.5 (128 MB) for my virtual memory. I will defragment the drive, then set it up in Control Panel | System | Performance | Virtual Memory

For the rest of the evening, I just want to organize my notes for the Physics exam. I will then work on the SS Tuesday evening after the OST exam.

D:\ copy win386.swp H:\

Virtual Cache: System.ini [vcache]

Min File Cache = ∅

Max File Cache = ~~4096,~~ 32768.

21:00 hrs... I better get busy with Physics. I am hitting a "wall".

1999 354 Mo 20 December 00:00 hrs

Julian Day: 2451532.7

Organism is currently "exactly" 12,000 days old. Also, it would have been my grandfathers (Hentrich's) birthday. I am so discouraged with Rutgers that I don't give a fuck about Physics or the Physics II exam. NJ IT? in Newark? man... fuck. I am lapsing into a deep melancholia. My nephew's computer usurped massive amounts of my time these past 3 days. I am fortunate to have completed the Java exam. I am so discouraged. What can I do? I have to focus on the final exams at BCC; but I also had better stay focused on Rutgers. I will call DVR and Rutgers Monday, Tuesday, and Wednesday. I will just wing the final exams. Fuck it. Two tears in a bucket

Fragile eggshell equilibrium. Peacefulness and serenity and general enthusiasm is instantly transformed into twisted, hateful bitterness. Is my happiness that superficial? These have not been wasted years.

I refuse to become suicidal (until I know for sure that I cannot begin studying Computer Science and Mathematics at Rutgers on Jan 18th). Then I can be suicidal. What can I do at this point but turn to poetry, philosophy, psychology? Not religion. My nephew got his machine back. I am exhausted.

I know he appreciates it, just as he knows I will appreciate a 4.3 GByte harddrive — or even an 8.4 GByte harddrive — to serve as slave to my 10GByte.

Will any of this matter in January if this "lack of an acceptance letter from Rutgers" problem is not soon resolved? I have done all I can do: straight A's full time for 18 months. I could not have done any better than this.

Now I will try to sleep for a couple hours before my father picks me up for work. What would I do without my parents? If I go to Rutgers and live in New Brunswick, I would feel more independent. I would be forced to be independent. Melancholia is good. Suddenly I am aware of the superficiality of happiness.

I am happy if and only if this. I am happy as long as NOT that. I bow my head in shame for I am so dependent upon events outside my control to maintain my sanity.

My sadness is pathetic. The world is a tough place. Poor baby. Cry in your pillow. I will not miss life when it leaves me; but I did not "come into this world". I "came out of" this world.

I love my smell after 3 days without bathing. Funk is real. Sweet fragrance is superficial and fake. Happy man when accepted into University. Suicidal mental patient when things go so differently than he had planned. Irrational Demand: I must start at Rutgers on January 18th 2000. Life is unfair if I dont go to Rutgers.

I deserve to have my tuition paid for. If I dont go to Rutgers, I will apply for a job with a big company and ask to be sent to NJIT, or I will hope to die in my sleep or just HOPE TO DIE!

03:00 hrs I think I will use TECHNICAL LOGBOOKS when preparing for exams. This is how I will review key ideas. Strange, though, now that it is out of reach (at the other residence) I feel inclined to make a "religious-like" cogitation to get me moving, to set the tone for this afternoon when I wake up in the basement.

I will drink coffee all day.

COGITATION #Ø61 Inverse Matrices

if A is invertible, there exists a unique A^{-1} such that $AA^{-1} = A^{-1}A = I$

$$A\vec{x} = \vec{b} \; : \; (AA^{-1})\vec{x} = A^{-1}\vec{b}$$

$$I\,\vec{x} = A^{-1}\vec{b}$$

solution: $\vec{x} = A^{-1}\vec{b}$

$AA^{-1} = I$ is the set of the 3 systems:

$$A\vec{x}_1 = \vec{e}_1 \;,\quad A\vec{x}_2 = \vec{e}_2 \;,\quad A\vec{x}_3 = \vec{e}_3$$

Finding the three solutions $\vec{x}_1, \vec{x}_2, \vec{x}_3$ gives A^{-1}.

We solve all 3 systems at once.

Form augmented matrix $[A \; I]$ and use elimination, row exchanges; multiplication of rows by a scalar to produce $[I \; A^{-1}]$

In other words, multiply $[A \; I]$ by A^{-1} to produce $[I \; A^{-1}]$

An example is called for.

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Compute A^{-1} where $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$

$$[A \;\; I] = \begin{bmatrix} 2 & -1 & 0 & 1 & 0 & 0 \\ -1 & 2 & -1 & 0 & 1 & 0 \\ 0 & -1 & 2 & 0 & 0 & 1 \end{bmatrix}$$

$\text{pivot}_1 = 2$; get rid of nonzero coefficients under the 2. Add $\frac{1}{2} r_1$ to r_2 to get r_2

my notation: $r_2 \longleftarrow \frac{r_1}{2} + r_2$

"is replaced by"

$$\begin{bmatrix} 2 & -1 & 0 & 1 & 0 & 0 \\ 0 & \frac{3}{2} & -1 & \frac{1}{2} & 1 & 0 \\ 0 & 0 & \frac{4}{3} & \frac{1}{3} & \frac{2}{3} & 1 \end{bmatrix} \qquad r_3 \longleftarrow \frac{2r_2}{3} + r_3$$

$-\frac{2}{3} + \frac{6}{3} = \frac{4}{3}$

We now use elimination to place ZEROS above the pivots. hence, $r_2 \longleftarrow \frac{3r_3}{4} + r_2$ and $r_1 \longleftarrow \frac{2r_2}{3} + r_1$

$$\begin{bmatrix} 2 & 0 & 0 & \frac{3}{2} & 1 & \frac{1}{2} \\ 0 & \frac{3}{2} & 0 & \frac{3}{4} & \frac{3}{2} & \frac{3}{4} \\ 0 & 0 & \frac{4}{3} & \frac{1}{3} & \frac{2}{3} & 1 \end{bmatrix}$$

$\frac{3}{12} + \frac{1}{2} = \frac{3}{12} + \frac{6}{12} = \frac{9}{12} = \frac{3}{4}$

$\frac{6}{12} + \frac{12}{12} = \frac{18}{12} = \frac{3}{2}$

Then multiply by inverse of pivots (divide rows by row pivot)

$\frac{r_1}{2}$, $\frac{2r_2}{3}$, $\frac{3r_3}{4}$

$$\left[\underbrace{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}}_{I} \;\; \underbrace{\begin{matrix} \frac{3}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} & \frac{3}{4} \end{matrix}}_{A^{-1}} \right]$$

All this to produce $A^{-1} = \begin{bmatrix} 3/4 & 1/2 & 1/4 \\ 1/2 & 1 & 1/2 \\ 1/4 & 1/2 & 3/4 \end{bmatrix}$

COGITATION #062 LU Factorization

Lower Δ Matrix

Upper Δ Matrix

Given system $A\vec{x} = \vec{b}$, write $A = LU$

Then solve $L\vec{c} = \vec{b}$ and then solve $U\vec{x} = \vec{c}$

$\underbrace{LU}_{A}\vec{x} = \vec{b}$ $\quad L\underbrace{U\vec{x}}_{\vec{c}} = \vec{b}$

Suppose $A\vec{x} = \vec{b}$ Rewrite as $LU\vec{x} = \vec{b}$

let $U\vec{x} = \vec{c}$, then solve $L\vec{c} = \vec{b}$

and then solve $U\vec{x} = \vec{c}$

This gives $\vec{x}$, solution to $A\vec{x} = \vec{b}$

$$\left.\begin{aligned} 2x + y \quad &= 2 \\ x + 2y + z &= 4 \\ y + 2z &= 6 \end{aligned}\right\} \text{gives } A\vec{x} = \vec{b}$$

where $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$, $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$

① $L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ 0 & \frac{2}{3} & 1 \end{bmatrix}$ $\frac{3}{2}$ 1 $\frac{2}{3}$ of eliminations E_{21}, E_{32}

solve $L\vec{c} = \vec{b}$

where $\vec{c} = [c_1, c_2, c_3]$

$\vec{c} = [2, 3, 4]$

$c_1 = 2$

$\frac{c_1}{2} + c_2 = 4$ $\quad = \vec{b}$ $\quad c_2 = 3$

$\frac{2c_2}{3} + c_3 = 6$ $\quad c_3 = 4$

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② $U = \begin{bmatrix} 2 & 1 & 0 \\ & & \\ & & \end{bmatrix}$ A_{r_1} perform elimination on A

$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$

L

How did we obtain U?

L is a combination of inverse elimination matrices.

L^{-1} is $E_1 E_2 E_n$ when L is $E_n^{-1} E_2^{-1} E_1^{-1}$

So we obtain U by "total E_n matrices" times A.

Try it $E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $E_{21}A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & \frac{3}{2} & 1 \\ 0 & 1 & 2 \end{bmatrix}$

$E_{32} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{2}{3} & 1 \end{bmatrix}$ and $E_{32}A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & \frac{3}{2} & 0 \\ 0 & 0 & \frac{4}{3} \end{bmatrix}$

But $E_{32}E_{21}A = U$!

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2/3 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ 1/3 & -\frac{2}{3} & 1 \end{bmatrix}$$

$$E_{32}E_{21}A = \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ 1/3 & -2/3 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 & 0 \\ 0 & 3/2 & 1 \\ 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} & & 0 \\ & & \\ & & \end{bmatrix}$$

$$E_{21}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad E_{32}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2/3 & 0 \end{bmatrix}$$

~~UUF~~ Confusion - Time to Sleep.